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FINITE-AMPLITUDE WAVES IN A HOMOGENEOUS FLUID WITH A FLOATING ELASTIC PLATE

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Equations for three nonlinear approximations of a wave perturbation in a homogeneous ideal incompressible fluid covered by a thin elastic plate are obtained using the method of multiple scales and taking into account that the acceleration of vertical flexural displacements of the plate is nonlinear. Based on the obtained equations, asymptotic expansions up third-order terms are constructed for the fluid velocity potential and the perturbations of the plate–fluid interface (plate bending) caused by a traveling periodic wave of finite amplitude. The wave characteristics are analyzed as functions of the elastic modulus and thickness of the plate and the length and tilt of the initial fundamental harmonic wave.

Key words: *surface waves, flexural-gravity waves, floating plate vibrations.*

Introduction. Wave perturbations in a fluid with a floating elastic plate in a linear formulation were studied in [1–7]. Finite-amplitude waves in a homogeneous fluid with an elastic ice plate neglecting the nonlinearity of vertical displacements of the plate were investigated in [8, 9]. In [10], the characteristics of progressive surface finite-amplitude waves were analyzed as functions of the thickness and nonlinearity of vertical vibrations of an absolutely flexible plate (broken ice). In the present work, the effect of a floating elastic plate on the propagation of periodic finite-amplitude waves is studied using the method of multiscale asymptotic expansion and taking into account that the acceleration of vertical flexural displacements of the plate is nonlinear.

Formulation of the Problem. Let a thin elastic plate float on the surface of a homogeneous ideal incompressible fluid filling an unbounded pool of constant depth H. Assuming that the fluid motion is potential and the plate performs vibrations without separation, we examine the effect of the elasticity and thickness of the plate on the propagation of finite-amplitude flexural-gravity waves. In the dimensionless variables $x = kx_1$, $z = kz_1$, and $t = \sqrt{kg}\,t_1$ (k is the wavenumber), the problem reduces to solving the Laplace equation

$$\Delta\varphi = 0, \qquad -\infty < x < \infty, \quad -H \leqslant z \leqslant \zeta \tag{1}$$

for the velocity potential $\varphi(x, z, t)$ with the following boundary conditions:

$$D_1 k^4 \frac{\partial^4 \zeta}{\partial x^4} + \varkappa k \frac{\partial}{\partial z}\Big[\frac{1}{2}\Big(\frac{\partial\varphi}{\partial x}\Big)^2 - \frac{\partial\varphi}{\partial t}\Big] + \zeta - \frac{\partial\varphi}{\partial t} + \frac{1}{2}\Big[\Big(\frac{\partial\varphi}{\partial x}\Big)^2 + \Big(\frac{\partial\varphi}{\partial z}\Big)^2\Big] = 0 \tag{2}$$

on the plate–fluid interface ($z = \zeta$) and

$$\frac{\partial\varphi}{\partial z} = 0 \tag{3}$$

at the bottom of the pool ($z = -H$); the initial conditions are

$$\zeta = f(x), \qquad \frac{\partial\zeta}{\partial t} = 0. \tag{4}$$

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Here

$$D_1 = \frac{D}{\rho g}, \qquad D = \frac{Eh^3}{12(1-\nu^2)}, \qquad \varkappa = h\,\frac{\rho_1}{\rho},$$

E, h, ρ_1, and ν are the normal elasticity modulus, thickness, density, and Poisson ratio of the plate, respectively, ρ is the fluid density, and g is the acceleration due to gravity. The velocity potential $\varphi(x,z,t)$ and the perturbation of the plate–fluid interface $\zeta(x,t)$ are related by the kinematic condition

$$\frac{\partial\zeta}{\partial t} - \frac{\partial\zeta}{\partial x}\,\frac{\partial\varphi}{\partial x} + \frac{\partial\varphi}{\partial z} = 0. \tag{5}$$

In the dynamic condition (2), the expression containing the factor $\varkappa$ describes the inertia of vertical displacements of the plate; the first term is due to the nonlinearity of vertical flexural displacements of the plate.

Equations for Nonlinear Approximations. Under the assumption of the validity of the expansions

$$\zeta = \varepsilon\zeta_0, \qquad \varphi = \varepsilon\varphi_0, \qquad f = \varepsilon f_0,$$

$$\zeta_0 = \zeta_1 + \varepsilon\zeta_2 + \varepsilon^2\zeta_3 + O(\varepsilon^3), \qquad \varphi_0 = \varphi_1 + \varepsilon\varphi_2 + \varepsilon^2\varphi_3 + O(\varepsilon^3),$$

$$f_0 = f_1 + \varepsilon f_2 + \varepsilon^2 f_3 + O(\varepsilon^3),$$

problem (1)–(5) is reduced by the method of multiple scales [11] to determining nonlinear approximations of order ε^n from the following equations:

$$\Delta\varphi_n = 0, \qquad -\infty < x < \infty, \quad -H \leqslant z \leqslant 0; \tag{6}$$

$$D_1 k^4\,\frac{\partial^4\zeta_n}{\partial x^4} - \varkappa k\,\frac{\partial^2\varphi_n}{\partial z\,\partial T_0} + \zeta_n - \frac{\partial\varphi_n}{\partial T_0} = F_n^*, \qquad z = 0; \tag{7}$$

$$\frac{\partial\zeta_n}{\partial T_0} + \frac{\partial\varphi_n}{\partial z} = L_n, \qquad z = 0; \tag{8}$$

$$\frac{\partial\varphi_n}{\partial z} = 0, \qquad z = -H; \tag{9}$$

$$\zeta_n = f_n(x), \qquad \frac{\partial\zeta_n}{\partial T_0} = G_n, \qquad t = 0. \tag{10}$$

Here $f_n(x)$ is the approximation of order ε^n for the initial displacement of the plate–fluid interface;

$$T_0 = t, \qquad T_1 = \varepsilon t, \qquad T_2 = \varepsilon^2 t,$$

$$F_n^* = F_n + F_n^0, \qquad F_1 = F_1^0 = L_1 = G_1 = 0, \qquad n = 1,2,3,$$

$$F_2 = \zeta_1\,\frac{\partial^2\varphi_1}{\partial T_0\,\partial z} + \frac{\partial\varphi_1}{\partial T_1} + \frac{\partial\zeta_1}{\partial T_0}\,\frac{\partial\varphi_1}{\partial z} - \frac{1}{2}\Big[\Big(\frac{\partial\varphi_1}{\partial x}\Big)^2 + \Big(\frac{\partial\varphi_1}{\partial z}\Big)^2\Big] + \varkappa k N,$$

$$N = \frac{\partial\zeta_1}{\partial T_0}\,\frac{\partial^2\varphi_1}{\partial z^2} + \zeta_1\,\frac{\partial^3\varphi_1}{\partial T_0\,\partial z^2} + \frac{\partial^2\varphi_1}{\partial T_1\partial z}, \qquad L_2 = \frac{\partial\zeta_1}{\partial x}\,\frac{\partial\varphi_1}{\partial x} - \zeta_1\,\frac{\partial^2\varphi_1}{\partial z^2} - \frac{\partial\zeta_1}{\partial T_1}, \qquad G_2 = -\frac{\partial\zeta_1}{\partial T_1},$$

$$F_3 = \zeta_1 N_1 + \frac{1}{2}\,\zeta_1^2\,\frac{\partial^3\varphi_1}{\partial T_0\,\partial z^2} + \zeta_2\,\frac{\partial^2\varphi_1}{\partial T_0\partial z} + \frac{\partial\varphi_2}{\partial T_1} + \frac{\partial\varphi_1}{\partial T_2} - \frac{\partial\varphi_1}{\partial x}\,\frac{\partial\varphi_2}{\partial x} - \frac{\partial\varphi_1}{\partial z}\,\frac{\partial\varphi_2}{\partial z} + N_2 + \varkappa k N_3,$$

$$N_1 = \frac{\partial^2\varphi_1}{\partial T_1\,\partial z} + \frac{\partial^2\varphi_2}{\partial T_0\,\partial z} - \frac{\partial^2\varphi_1}{\partial z\,\partial x}\,\frac{\partial\varphi_1}{\partial x} - \frac{\partial^2\varphi_1}{\partial z^2}\,\frac{\partial\varphi_1}{\partial z},$$

$$N_2 = \frac{\partial\zeta_1}{\partial T_0}\,\frac{\partial\varphi_2}{\partial z} + \frac{\partial\zeta_2}{\partial T_0}\,\frac{\partial\varphi_1}{\partial z} + \zeta_1\,\frac{\partial\zeta_1}{\partial T_0}\,\frac{\partial^2\varphi_1}{\partial z^2} + \frac{\partial\zeta_1}{\partial T_1}\,\frac{\partial\varphi_1}{\partial z} - \frac{\partial\zeta_1}{\partial x}\,\frac{\partial\varphi_1}{\partial z}\,\frac{\partial\varphi_1}{\partial x},$$

$$N_3 = \zeta_1 N_4 + \frac{1}{2}\,\zeta_1^2\,\frac{\partial^4\varphi_1}{\partial T_0\,\partial z^3} + \zeta_2\,\frac{\partial^3\varphi_1}{\partial T_0\,\partial z^2} + \frac{\partial\zeta_1}{\partial T_0}\,\frac{\partial^2\varphi_2}{\partial z^2} + \frac{\partial\zeta_2}{\partial T_0}\,\frac{\partial^2\varphi_1}{\partial z^2} + \frac{\partial^2\varphi_2}{\partial z\,\partial T_1} + \frac{\partial\zeta_1}{\partial T_1}\,\frac{\partial^2\varphi_1}{\partial z^2} + \frac{\partial^2\varphi_1}{\partial z\,\partial T_2},$$

$$N_4 = \frac{\partial^3 \varphi_2}{\partial T_0\, \partial z^2} + \frac{\partial \zeta_1}{\partial T_0}\, \frac{\partial^3 \varphi_1}{\partial z^3} + \frac{\partial^3 \varphi_1}{\partial z^2\, \partial T_1}, \qquad G_3 = -\frac{\partial \zeta_1}{\partial T_2} - \frac{\partial \zeta_2}{\partial T_1},$$

$$L_3 = \zeta_1 N_5 - \frac{1}{2}\,\zeta_1^2\, \frac{\partial^3 \varphi_1}{\partial z^3} - \zeta_2\, \frac{\partial^2 \varphi_1}{\partial z^2} + \frac{\partial \zeta_2}{\partial x}\, \frac{\partial \varphi_1}{\partial x} + \frac{\partial \zeta_1}{\partial x}\, \frac{\partial \varphi_2}{\partial x} - \frac{\partial \zeta_2}{\partial T_1} - \frac{\partial \zeta_1}{\partial T_2} + \frac{\partial \varphi_1}{\partial z}\Big(\frac{\partial \zeta_1}{\partial x}\Big)^2,$$

$$N_5 = \frac{\partial \zeta_1}{\partial x}\, \frac{\partial^2 \varphi_1}{\partial z\, \partial x} - \frac{\partial^2 \varphi_2}{\partial z^2}, \qquad F_2^0 = -\varkappa k\, \frac{\partial^2 \varphi_1}{\partial x\, \partial z}\, \frac{\partial \varphi_1}{\partial x},$$

$$F_3^0 = -\varkappa k\Big[\zeta_1 N_6 + \frac{\partial^2 \varphi_1}{\partial x\, \partial z}\Big(\frac{\partial \varphi_2}{\partial x} + \frac{\partial \zeta_1}{\partial x}\, \frac{\partial \varphi_1}{\partial z}\Big) + \frac{\partial \varphi_1}{\partial x}\Big(\frac{\partial^2 \varphi_2}{\partial x\, \partial z} + \frac{\partial \zeta_1}{\partial x}\, \frac{\partial^2 \varphi_1}{\partial z^2}\Big)\Big],$$

$$N_6 = \Big(\frac{\partial^2 \varphi_1}{\partial x\, \partial z}\Big)^2 + \frac{\partial \varphi_1}{\partial x}\, \frac{\partial^3 \varphi_1}{\partial x\, \partial z^2}.$$

We note that the presence of the terms F_2^0 and F_3^0 on the right sides of the dynamic conditions (7) for the second approximation ($n = 2$) and third approximation ($n = 3$) is due to the nonlinearity of vertical displacements of the plate.

Expressions for the Velocity Potential and Perturbations of the Plate–Fluid Interface. Equations (6)–(10) are obtained for the general case of unsteady finite-amplitude perturbations. In the case of traveling periodic waves, we seek a solution of relations (6)–(10) by specifying $f_n(x)$ in appropriate form. For this, the first approximation ($n = 1$) of the perturbation of the plate–fluid interface ζ_1 is represented in the form of a wave

$$\zeta_1 = \cos\theta, \qquad \theta = x + \tau T_0 + \beta(T_1, T_2), \tag{11}$$

which moves opposite to the x direction. Then, from the kinematic condition (8), we obtain

$$\frac{\partial \varphi_1}{\partial z} = \tau \sin\theta, \qquad z = 0. \tag{12}$$

Satisfying boundary condition (9) on the bottom of the pool, we write the velocity potential φ_1 as

$$\varphi_1 = b_0 \cosh\,(z + H)\, \sin\theta. \tag{13}$$

Substitution of relation (13) into (12) yields $b_0 = \tau(\sinh\, H)^{-1}$. Hence,

$$\varphi_1 = b_1 \sin\theta, \qquad b_1 = \tau(\sinh\, H)^{-1} \cosh\,(z + H). \tag{14}$$

Taking into account (11) and (14), from the dynamic condition (7) we obtain the dispersion relation

$$\tau^2 = (1 + D_1 k^4)(1 + \varkappa k \tanh\, kH)^{-1} \tanh\, kH.$$

Substitution of (11) and (14) into the right sides of expressions (7) and (8), subject to the condition of no fundamental harmonic, yields

$$\zeta_2 = a_2 \cos 2\theta, \qquad \varphi_2 = b_2 \sin 2\theta, \tag{15}$$

where

$$a_2 = 3\tau^2 \eta_2/(4\mu_2 \tanh\, H), \qquad b_2 = \tau\nu_2 \cosh\, 2(z + H)/(4\mu_2 \cosh\, 2H \tanh\, H),$$

$$\eta_2 = (\tanh\, H - \coth\, H - 2\varkappa k) \tanh\, 2H, \qquad \nu_2 = \tau^2(5 \tanh\, H - \coth\, H + 2\varkappa k) - 2(1 + 16 D_1 k^4),$$

$$\mu_2 = (1 + 16 D_1 k^4) \tanh\, 2H - 2\tau^2(1 + 2\varkappa k \tanh\, 2H).$$

Substituting the solution (11), (14) for the first approximation and the solution (15) for the second approximation into the right sides of the dynamic condition (7) and kinematic condition (8) of the problem for the third ($n = 3$) approximation and eliminating the secularity-producing terms, we obtain

$$\zeta_3 = a_3 \cos 3\theta, \qquad \varphi_3 = b_3 \sin 3\theta,$$

$$a_3 = \tau^2 \eta_3/\mu_3, \qquad b_3 = \tau\nu_3 \cosh\, 3(z + H)/(3\mu_3 \cosh\, 3H),$$

$$\eta_3 = (l_1 - 3\varkappa k l_2) \tanh\, 3H - l_2, \qquad \nu_3 = 3\tau^2 l_1 - l_2(1 + 81 D_1 k^4),$$

$$l_1 = \varkappa k l_{11} + l_{12}, \qquad l_2 = (1/2)a_2(3\coth H + 6\coth 2H) - (3/2)\coth H \coth 2H + 5/8,$$

$$l_{11} = a_2(5\coth 2H - (1/2)\coth H) + (1/2)(\coth H - 5\coth 2H)\coth H - 1/8,$$

$$l_{12} = a_2(11/2 - \coth H \coth 2H) + ((1/2)\coth H \coth 2H - 15/8)\coth H,$$

$$\mu_3 = (1 + 81 D_1 k^4)\tanh 3H - 3\tau^2(1 + 3\varkappa k \tanh 3H),$$

$$\beta = \tau\sigma_0 T_2, \qquad \sigma_0 = (1/2)[l_3 - l_4(\varkappa k + \coth H)^{-1}],$$

$$l_3 = (1/2)a_2(\coth H + 2\coth 2H) - (1/2)\coth H \coth 2H - 3/8,$$

$$l_4 = \varkappa k l_{41} - l_{42}, \qquad l_{41} = a_2(\coth 2H - (5/2)\coth H) + (1/2)(\coth H - \coth 2H)\coth H - 3/8,$$

$$l_{42} = a_2(1/2 + \coth H \coth 2H) - (1/2)(\coth H \coth 2H - 5/4)\coth H.$$

As a result, the perturbation of the plate–fluid interface ζ and the fluid velocity potential φ up to third-order terms are determined from the expressions

$$\zeta = \sum_{n=1}^{3} \varepsilon^n a_n \cos n\theta, \qquad \varphi = \sum_{n=1}^{3} \varepsilon^n b_n \sin n\theta,$$

$$\theta = x + \sigma t, \qquad \sigma = \tau(1 + \varepsilon^2\sigma_0), \qquad a_1 = 1.$$

In the dimensional variables $\zeta = \zeta/k$, $\varphi = \varphi\sqrt{kg}/k^2$, and $\varepsilon = ak$, where a is the amplitude of the initial harmonic, we have

$$\zeta = a\cos\theta + a^2 k a_2 \cos 2\theta + a^3 k^2 a_3 \cos 3\theta,$$
$$\varphi = a\sqrt{g/k}\, b_1 \sin\theta + a^2\sqrt{kg}\, b_2 \sin 2\theta + a^3 k\sqrt{kg}\, b_3 \sin 3\theta, \qquad (16)$$

where

$$\theta = kx + \sigma_1(1 + \sigma^0)t, \qquad \sigma^0 = a^2k^2\sigma_0, \qquad \sigma_1 = \tau\sqrt{kg},$$

$$\tau^2 = (1 + D_1k^4)(1 + \varkappa k \tanh kH)^{-1}\tanh kH.$$

The phase velocity of the wave perturbations is determined from the formula

$$v = \tau\sqrt{g/k}\,(1 + \sigma^0).$$

In the short-wave approximation ($kH \gg 1$), the solution is simplified since the coefficients in (16) have the form

$$a_2 = \frac{3\varkappa k}{2\Delta_2}(1 + D_1k^4), \qquad b_2 = -\frac{1}{2\Delta_2}\, e^{2kz}\sqrt{\frac{1 + D_1k^4}{1 + \varkappa k}}\,[1 - (14 + 15\varkappa k)D_1k^4],$$

$$a_3 = [1 + 2\varkappa k + 24(\varkappa k)^2 - AD_1k^4 - A_1(D_1k^4)^2]/(4\Delta_2\Delta_3),$$

$$b_3 = e^{3kz}\sqrt{(1 + D_1k^4)/(1 + \varkappa k)}\,[13 + 7\varkappa k + 12(\varkappa k)^2 - BD_1k^4 + 2B_1(D_1k^4)^2]/(24\Delta_2\Delta_3),$$

$$A = 13 - 4\varkappa k - 63(\varkappa k)^2, \qquad A_1 = 14 - 2\varkappa k - 39(\varkappa k)^2,$$

$$B = 499 - 529(\varkappa k) - 1014(\varkappa k)^2, \qquad B_1 = 1899 + 4461\varkappa k + 2601(\varkappa k)^2,$$

$$\Delta_2 = 1 + 3\varkappa k - 2(7 + 6\varkappa k)D_1k^4, \qquad \Delta_3 = 1 + 4\varkappa k - 3(13 + 12\varkappa k)D_1k^4,$$

$$\sigma_0 = -\{3 - 7\varkappa k - 12(\varkappa k)^2 - 2[21 + 41\varkappa k + 21(\varkappa k)^2]D_1k^4\}/[8(1 + \varkappa k)\Delta_2].$$

Analysis of Results. Solution (16) is valid outside the small regions of the resonant wavenumbers $k = k_1$ and $k = k_2$ which are positive real roots of the equations $\mu_2 = 0$ and $\mu_3 = 0$, respectively. These equations remain valid if the nonlinearity of vertical displacements of the plate is neglected. For $kH \gg 1$, the values of k_1 and k_2

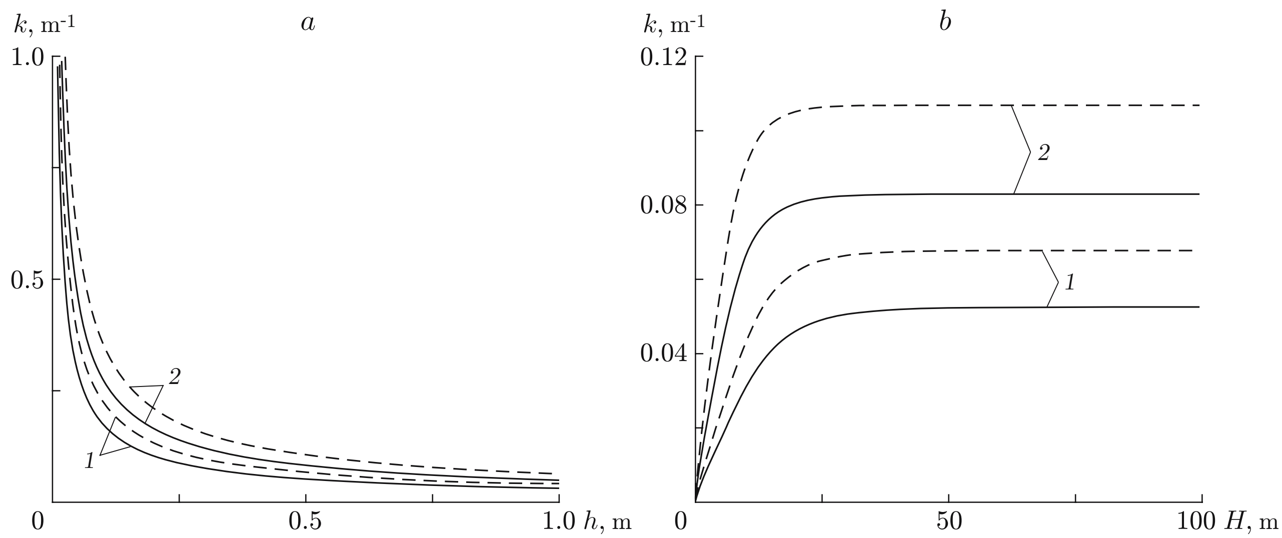

Fig. 1. Resonant wavenumber versus plate thickness (a) and pool depth (b) for $\rho_1/\rho = 0.87$, $\nu = 0.34$, and $E = 3 \cdot 10^9$ (1) and $5 \cdot 10^8$ N/m^2 (2): solid curves correspond to k_2 and dashed curves to k_1.

satisfy the equations $\Delta_2 = 0$ and $\Delta_3 = 0$. From this, we find the estimates $k_1 < \sqrt[4]{1/(4D_1)}$ and $k_2 < \sqrt[4]{1/(9D_1)}$ ($k_2 < k_1$).

Figure 1 shows curves of the resonant wavenumbers versus ice plate thickness at $H = 100$ m (Fig. 1a) and fluid layer depth at $h = 0.5$ m (Fig. 1b) for $\rho_1/\rho = 0.87$ and $\nu = 0.34$. In Fig. 1, it is evident that, for a fixed depth of the fluid layer, the values of k_1 and k_2 increase with decreasing cylindrical rigidity of the plate, tending to infinity in a fluid with open surface ($h = 0$) and in a fluid covered by an absolutely flexible ($h \neq 0$, $E = 0$) plate (broken ice). The effect of the fluid layer depth is manifested in an increase in the values of k_1 and k_2. However, as the rigidity of the plate decreases, the depth H at which this effect is significant also decreases.

To evaluate the effect of the thickness and elastic modulus of the floating ice plate on the amplitude–phase characteristics of the wave perturbation, we performed numerical calculations for the same values of the parameters E, ν, and ρ_1/ρ as in Fig. 1.

An analysis of the calculation results shows that the effect of the cylindrical rigidity of an ice plate on the perturbation structure is determined by the depth of the fluid layer H, the length $\lambda = 2\pi/k$, and the tilt $\varepsilon = ak$ of the initial fundamental harmonic wave (Fig. 2). It is evident that the elasticity of the plate can influence not only the vibration amplitude, by decreasing it, but also the wave profile. In the short-wave range (see Fig. 2b), an increase in the elasticity of the plate leads an increase in the effect of higher harmonics. In addition, accounting for the nonlinearity of acceleration of vertical displacements leads to a vibration phase shift along the direction of motion of the wave if the plate is absolutely flexible ($E = 0$) and to a vibration-phase lag in the case of an elastic plate. A quantitative estimate of the change in the vibration phase due to the nonlinearity of the acceleration can be obtained from Figs. 3 and 4, which show the distributions of the vibration-phase shift $|\sigma^0|$ and the phase velocity v, respectively ($E = 3 \cdot 10^9$ N/m^2). It should be noted that the curves in Figs. 3 and 4 are given only for the flexural branch of the dispersion curve of the phase velocity. In the gravity branch, where the gravity is greater than the elasticity force, the effect of the acceleration nonlinearity is qualitatively the same as in the case of an absolutely flexible plate (broken ice) studied in [10]. In the flexural-gravity region of the dispersion curve, the phase velocity can both increase and decrease due to the nonlinearity of vertical accelerations.

Conclusions. Partial differential equations describing three nonlinear approximations of solutions of the time–space evolution problem for an arbitrary initial finite-amplitude perturbation of a plate–fluid interface (plate bending) were derived based on the equations of nonlinear-wave dynamics in a homogeneous ideal incompressible fluid with a floating thin elastic plate and using the method of multiscale asymptotic expansions. In the case of traveling periodic waves for the fluid velocity potential and deviation of the plate–fluid interface from the unperturbed state, solutions of these equations are constructed in the form of asymptotic series up to third-order terms.

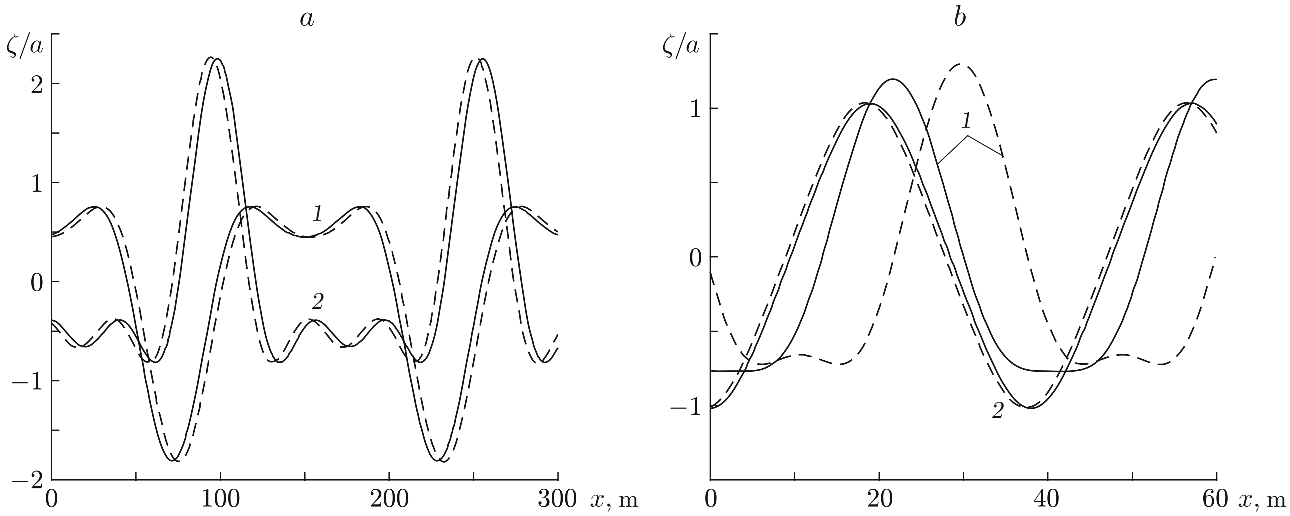

Fig. 2. Perturbation profiles of the plate–fluid interface along the x axis for $E = 0$ (curves 2) and $3 \cdot 10^9$ N/m^2 (curves 1): (a) $H = 10$ m, $h = 0.7$ m, $k = 0.04$ m^{-1}, $t = 150$ sec, and $\varepsilon = 0.08$; (b) $H = 100$ m, $h = 0.15$ m, $k = 0.165$ m^{-1}, $t = 70$ sec, and $\varepsilon = 0.33$; solid curves show the results obtained neglecting the nonlinearity of plate vertical displacements and dashed curves show the results obtained taking into account the nonlinearity of vertical displacements of the plate.

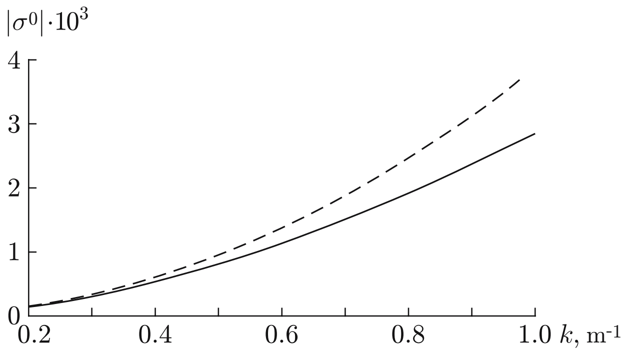

Fig. 3

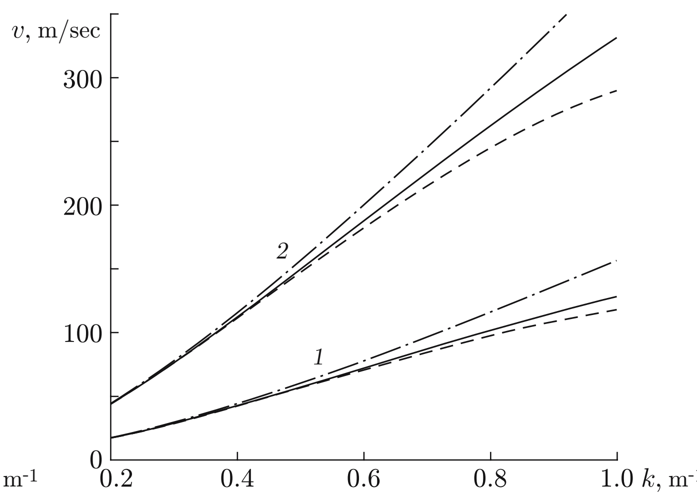

Fig. 4

Fig. 3. Vibration phase shift versus wavenumber at $h = 0.5$ m: solid curves shows the results obtained neglecting the nonlinearity of vertical displacement of the plate, dashed curves show the results taking into account the nonlinearity of vertical displacements of the plate.

Fig. 4. Phase velocity versus wavenumber for plate thicknesses $h = 0.5$ (curves1) and 1.0 m (curves 2): solid curves are the results neglecting the nonlinearity of vertical displacements of the plate, dashed curves are the results taking into account the nonlinearity of vertical displacement of the plate, and dash-and-dotted curves correspond to the fundamental linear harmonic.

The amplitude-phase characteristics of wave perturbations are analyzed as functions of the elastic modulus and thickness of the plate, length and tilt of the initial fundamental linear harmonic wave. It is shown that plate elasticity is responsible not only for a decrease in the flexural wave amplitude but also on the variation of the wave shape along the wave-propagation direction. In the short-wavelength range, the elasticity effect leads to an increase in the contribution of the high-order harmonics. Accounting for the nonlinearity of vertical displacements of an elastic plate leads to a vibration-phase lag, whereas in the case of an absolutely flexible plate, the reverse effect is observed.

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